Solutions

Ans1:

Matrix is not in Reduced Row Echelon Form because of the following reasons which are:

1. Row of all zeroes are not at the bottom.

Matrix is in Reduced Row Echelon Form because of the following reasons which are:

1. Row of all zeroes are at the bottom.
2. Zeroes must be above each pivot or pivots are only non-zero entry in each column which are clearly present in the matrix.

Matrix is not in Reduced Row Echelon Form because of the following reasons which are:

1. Zeroes must be above each pivot or pivots are only non-zero entry in each column.

Ans2:

The rank of matrix is because it contains 3 independent vectors.



Making it Reduced row echelon form,

is in reduced row echelon form.

The rank of matrix is because it contains independent vectors.

Ans3: The row space and column space have the same dimension

We know the rank of matrix is and its dimension is .

.

The dimensions for four fundamental subspaces are:

The row space, of matrix has dimension 5, equal to the rank.

The column space, of matrix also has dimension 5.

The null space, of matrix has dimension .

The left null space, of matrix has dimension

The sum of all four dimensions.

Ans4: We know the rank of matrix is and its dimension is .

.

The column space, of matrix also has dimension and a subspace of

The left null space, of matrix has dimension It means that the

left null space is just the zero vector.

Ans5:

is in reduced row echelon form.

Now, finding the null space vectors,

Suppose, if we take , then , , .

Therefore,

Ans6:

Making Augmented Matrix,

There are no solutions.

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Therefore, we get,

General Solutions for

Ans7: Projection of onto the column space defined by matrix ,



---------(1)

---------(2)

From equations (1) and (2),



----(3)

----(4)

From equations (3) and (4),

Ans8:

Transforming matrix into reduced row echelon form,

Now, is a matrix with pivot columns.

Matrix has linear independent vectors.

Also, column of matrix span in

Therefore, Matrix basis for the column space of

Now, using the Gram-Schmidt Process to find an orthogonal basis:

Let’s take .

Constructing , starting with and subtracting its projection along .

The three orthonormal vectors:

Ans9:

Calculating the eigenvalues and eigenvectors of matrix ,

--------(1)

Finding the determinant of equation (1),

The roots are

These are the eigenvalues.

Next, finding the eigenvectors,

For ,

Finding the reduced row echelon form of ,

Computing the null space of reduced row echelon form of matrix

Suppose, if we take then .

Therefore,

This is the null space.

Therefore, is the eigenvector.

For ,

Finding the reduced row echelon form of ,

Computing the null space of reduced row echelon form of matrix

Suppose, if we take then .

Therefore,

This is the null space.

Therefore, is the eigenvector.

Form the matrix whose column is eigenvector number : .

Form the diagonal matrix whose element are row column is eigenvalue number

Both matrices and are such that the initial matrix .

Ans10:

Finding the transpose of matrix ,

Now multiplying the matrix with its transpose ,

Now, finding both eigenvalues and eigenvectors of ,

--------(1)

Finding the determinant of equation (1),

The roots are

These are the eigenvalues.

Next, finding the eigenvectors,

For ,

Finding the reduced row echelon form of ,

Computing the null space of reduced row echelon form of matrix

Suppose, if we take then .

Therefore,

This is the null space.

Therefore, is the eigenvector.

For ,

Finding the reduced row echelon form of ,

Computing the null space of reduced row echelon form of matrix

Suppose, if we take then .

Therefore,

This is the null space.

Therefore, is the eigenvector.

The columns of the matrix is the normalized vectors:

The matrix is a zero matrix with on its diagonal:

Now,

Therefore, all three matrices are such way that the initial matrix

Ans11:

1. R(User1, Amelie) = UUser1.S.VT = 1.5
2. The Strength of concept 1 in SVD decomposition: 13.27
3. Average rating of Harry Potter: [[2.41666667]]
4. Casablanca has overall highest rating: 2.5833333333333335

Ans12:

(b) Total percentage of variance captured by the first 2 components of PCA: 99.99999999999999

(c) The strength of each of the two principal components are: 1.006711409395973 , 1.0067114093959741

(d) The magnitude of each of the two principal components are: 1.0136452424516673 , 1.0136452424516684